Rekha V.V.I. Questions for 2023 Examination

Answer of below mentioned V.V.I. questions are present in your Rekha Examination Guide and Guess Part-I Maths-1

Answer any six questions

1. (a) Define partially ordered and totally ordered set and also distinguish between them. **V. V. I.** 5,6

   (b) Define addition of cardinal numbers and prove that $\alpha + (\beta + y) = (\alpha + \beta) + y$, where α, β and γ are any three cardinal numbers. **V. V. I.** 9

2. Define domination of sets. Also state and prove Schroder - Bernstein theorem. **V. V. I.** 14

3. (a) Define ordinal number with example and prove that the operation of multiplication of ordinal numbers is not commutative. **V. V. I.** 16

   (b) State Zorn's Lemma and prove that Zorn's lemma implies the axiom of choice. **V. V. I.** 18

4. (a) Define a group with an example and prove that $(ab)^{-1} = b^{-1} a^{-1}$, $\forall$ a, b $\in$ G where G be a group. **V. V. I.** 19,20

   (b) Prove that a necesary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that a $\in$ H, b $\in$ H $\Rightarrow ab^{-1} \in$H, where b^{-1} is the inverse of b in G. **V. V. I.** 27

5. (a) Show that the sets S of all positive rationals forms a group under the binary operation * defined by $a * b = \frac{1}{2} ab$. 20

   (b) Show that in a group (G, O), the identity element is unique. 21

6. (a) If G is a group and a, b $\in$ G, then prove that $(ab)^2 = a^2b^2$ if and only if G is abelian. 25

   (b) Define centre of a group and show that centre of a group G in a subgroups of G. 36

7. (a) State and prove Cayley's Theorem. **V. V. I.** 29

   (b) Prove that every subgroup of a cyclic group is cyclic. **VVI** 32

   (c) Define Cyclic group and show that every cyclic group is an abelian group. 8

8. (a) Define Hermitian and skew-Hermitian matrices. Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix. **V. V. I.** 54

   (b) Find the inverse of the matrix (similar) : **V. V. I.** 46

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

9. (a) Definne inverse of a matrix. If A and B be two non-singular matries of the same order, then show that $(AB)^{-1} = B^{-1} A^{-1}$. ... 45,46

(b) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$$ 46

10. Define rank of a matrix and prove that the rank of the transpose of a matrix is the same as that of the original matrix. 51

11. (a) Show that the equations :

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

are consistent and solve them. **V. V. I.** 76

(b) Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them. **V. V. I.** 76

12. (a) Prove that in an equation with real coefficients, imaginary roots occur in conjugate pairs. **V. V. I.** 81

(b) Prove that every equation of nth degree has n roots and no more. **V. V. I.** 80

(c) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, are in G.P. 85

13. (a) If $\alpha, \beta, \gamma, \delta$ be the roots of equation $x^4 + px^3 + qx^2 + rx + 5 = 0$, then find the value of $\sum \frac{\alpha\beta}{\gamma^2}$. **V. V. I.** 91

(b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\beta + \gamma, \gamma + \alpha$ and $\alpha + \beta$ and hence find the value of $(\beta+\gamma)(\gamma + \alpha)(\alpha + \beta)$. **V. V. I.** 98

14. (a) Solve $x^3 - x^2 - 16x + 20 = 0$ by Cardon's method (similar). **V VI** 103

(b) Remove the second term of the equation : **V. V. I.**

$$x^4 + 8x^3 + x - 5 = 0$$ 102

MATHS - 1 (Hons.) (2022)

Answer any six questions

1. State and prove well-ordering theorem and deduce axiom of choice from well-ordering theorem. 15, 17
2. (a) Explain the concept of an ordinal number. 13
   (b) Define countable set. Prove that set of rational numbers is countable. 6, 7
3. (a) In a partially ordered set (X, $\leq$), prove that every non-empty subset of X bounded above has a least upper bound if and only if every non-empty subset of X bounded below has a greatest lower bound. 15
   (b) Prove that any well ordered set $(X, \leq)$ is totally ordered.
4. (a) Show that every group upto order 4 is Abelion. 23
   (b) Prove that the order of a cyclic group is equal to the order of its generator. 32
5. (a) State and prove Lagrange's theorem for a finite group. 28
   (b) State and prove fundamental theorem of homomorphism of groups. 37
6. (a) Prove that subset of even permutations is a permutation group forms a normal subgroup.
   (b) Prove that intersection of two normal subgroups is always normal. 35
7. (a) Reduce the following matrix into Echelon form and hence find its rank

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

   (b) Prove that a system of non-homogenous equations $Ax = B$ is consistent if and only if rank of the co-eff. Matrix A is equal to the rank of the augmented matrix [A | B]. 71
8. (a) If A and B are Hermitian Matrices, show that AB + BA is Hermitian and AB–BA is skew Hermitian. 54
   (b) Define symmetric and skew symmetric matrices. Prove that every square matrix can be expressed uniquely as a sum of a symmetric and skew symmetric matrix. 53
9. (a) Verify Cayley – Hamilton theorem for the matrix

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and use the result to obtain A^{-1}. 60

(b) Show that the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x = 4y + 7z = 30$$

are consistent and solve them. 75

10. (a) Solve by Ferrari's method $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

(b) Discuss Cardon's method of solving the cubic equation.

11. (a) Form the equation whose roots are the several values of P,

where $P = \frac{\alpha - \beta}{\alpha - \gamma}$ and α, β, γ are the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$.

(b) Show that the equation

$$\frac{A^2}{x-a} - \frac{B^2}{x-b} + \frac{C^2}{x-c} + \ldots + \frac{L^2}{x-l} = x - m$$

Where a, b, c are numbers all different from one another can not have an imaginary root. 83

12. (a) If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\, \alpha^2\beta$. 93

(b) Reduce the cubic equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ to the form $z^3 + 3Hz + b = 0$, where symbols have their usual meanings. 101

MATHS - 1 (Hons.) (2021)

1. (a) Define partially ordered and totally ordered set and also distinguish between them. 5, 6

(b) Prove that the set R of all real numbers is uncountable and also prove that any open interval on the real line R is numerically equivalent to R.

2. (a) Define domination of sets. Also state and prove Schroder - Bernstein theorem. 14

(b) If E is any set then prove that card P (E) = $2^{\text{Card E}}$ Where P(E) denotes the power set of E.

3. (a) State and prove comparability theorem for cardinals.

(b) Define ordinal number with example and prove that the operation of multiplication of ordinal numbers is not commutative. 16

4. (a) Prove that the set of all nth roots of unity forms a finite abelian group of order n with respect to multiplication.

(b) Prove that the $\underline{|n}$ permutations on n symbols, $\frac{1}{2}\underline{|n}$ are even permutations and $\frac{1}{2}\underline{|n}$ are odd permutations.

5. (a) Prove that the relation of isomorphism in the set of all groups is an equivalence relation.
(b) Show that the union of two subgroups is a subgroup iff one is contained in the other 29

6. (a) State and prove Cayley's Theorem. 32
(b) Prove that every subgroup of a cyclic group is cyclic.

7. (a) Define Hermitian and skew-Hermitian matrices. Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix. 54
(b) Find the inverse of the matrix : 46

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

8. (a) Define unitary matrix and show that $\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary if $a^2 + b^2 + c^2 + d^2 = 1$
(b) Define rank of a matrix and prove that the rank of the transpose of a matrix is the same as that of the original matrix. 51

9. (a) Reduce the following matrix to normal form and hence find its rank:

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

(b) Show that the equations : 76

$$5x + 3y + 7z = 4$$
$$3x + 26y + 2z = 9$$
$$7x + 2y + 10z = 5$$

are consistent and solve them.

10. (a) Prove that in an equation with real coefficients, imaginary roots occur in conjugate pairs. 81
(b) Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in A. P and hence solve the equation $x^2 - 12x^2 + 39x - 28 = 0$

11. (a) If α, β,γ be the roots of equation $x^3 + 3x + 9 = 0$, find the value of $\alpha^9 + \beta^9 + \gamma^9$.
(b) If α, β, γ, δ be the roots of equation $x^4 + px^3 + qx^2 + rx + 5 = 0$, then find the value of $\sum \frac{\alpha\beta}{\gamma^2}$. 51

12. (a) Solve $x^3 - x^2 - 16x + 20 = 0$ by Cardon's method. 103
(b) Discuss Euler's solution of the biquadratic equation.

MATHS - 1 (Hons.) (2020)

Answer any six questions

1. (a) Define countable and uncountable sets and prove that the set of rational numbers is countable. 6,7

   (b) Define addition of cardinal numbers and prove that $\alpha + (\beta + y) = (\alpha + \beta) + y$, where α, β and γ are any three cardinal numbers. 9

2. (a) If f is a similarity map of the well ordered set $(x, \leq)$ onto the subset $Y \subseteq X$ then prove that $x \leq f(x)$ for all $z \in X$.

   (b) Prove that any well ordered set $(x, \leq)$ is totally ordered.

3. State and prove the well ordering thorem and deduce axiom of choice from well ordering theorem. 15,17

4. (a) Define a group with an example and prove that $(ab)^{-1} = b^{-1} a^{-1}$, $\forall$ a, b $\in$ G where G be a group. 19,20

   (b) Prove that a necesary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H$, $b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G. 27

5. (a) State and prove Lagrange's theorem for a group. 28

   (b) If G is a group and a, b $\in$ G, then prove that $(ab)^2 = a^2b^2$ if and only if G is abelian. 25

6. (a) State and prove the fundamental theorem of homomorphism of groups. 37

   (b) Prove that the intersection of two normal subgroups of a group G is also a normal subgroup of G. 35

7. (a) Define symmetric and skew symmetric matrices. Prove that every square matrix can be expressed uniquely as a sum of a symmetric and skew-symmetric matrix. 53

   (b) If A and B are Hermitian matrices, show that AB + BA is Hermitian and AB–BA is skew Hemitian

8. (a) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 4 & 7 \\ 3 & 6 & 2 & 1 \\ 0 & 0 & 1 & 5 \end{bmatrix}$.

   (b) Prove that a system of non-homogenous equation AX = B is consistent if and only if rank of the coefficient matrix A is equal to the rank of the augmented matrix [AB]. 71

9. (a) Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them. 76

   (b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ and use the result to obtain A^{-1}. 60

10. (a) Prove that every equation of nth degree has n roots and no more. 80

    (b) Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, are in G.P. 85

11. (a) If α, β and γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\Sigma\alpha^2 \beta$. ... 93

(b) Reduce the cubic equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ to the form $z^3 + 3Hz + b = 0$, where symbols have their usual meanings. 101

12. (a) Discuss Cardon's method of solving the cubic equations.

(b) Solve by Ferrari's method $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$.

MATHS - 1 (Hons.) (2019)

Answer any six questions

1. (a) Distinguish between partially and totally ordered sets by constructing an example of a partially ordered set which is not totally ordered. 6

(b) Prove that for any partially ordered set $(X, \leq)$ the following statements are equivalent:

(i) Every non-empty subset of X which has an upper bound has a least upper bound.

(ii) Every non-empty subset of X has a lower bound has a greatest lower bound. 15

2. State and prove Schroeder - Bernstein theorem. 14

3. (a) State Zorn's Lemma and prove that Zorn's lemma implies the axiom of choice. 18

(b) Define sum ordinal numbers and show that the operation of addition of ordinal number is not commutative. 16

4. (a) Show that the sets S of all positive rationals forms a group under the binary operation * defined by $a * b = \frac{1}{2} ab$. 20

(b) Show that in a group (G, O), the identity element is unique. 21

5. (a) Define centre of a group and show that centre of a group G in a subgroups of G. 36

(b) Show that every subgroup of a cyclic group is cyclic. 32

6. (a) If f is a homomorphism of a group G into a group G' than show that :

(i) $f(e) = e'$, where e is the identity of G, and e' is the identity of G' 37

(ii) $f(a^{-1})$ -$[f(a)]^{-1}$ $\forall a \in G$ 7

(b) Define Cyclic group and show that every cyclic group is an abelian group. 8

7. (a) Definne inverse of a matrix. If A and B be two non-singular matries of the same order, then show that $(AB)^{-1} = B^{-1} A^{-1}$. 45,46

(b) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 9 \end{bmatrix}$$

.... 46

8. State and prove Cayley-Hamilton theorem. 60

9. Show that the equations.

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

are consistant and solve them. 75

10. (a) Show that in an equation with real co-efficlents, imaginary roots occur in conjugate pairs. 81

(b) Find the condition that the roots. $\alpha, \beta, \gamma, \delta$ of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ should be connected by the relation $\alpha\beta = \gamma\delta$. 86

11. (a) If α, β, γ are the roots of the cubic equation of $x^3 + px^2 + qx + r = 0$, find the value of $\Sigma\alpha^2 \beta^2$. 91

(b) If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $\beta + \gamma, \gamma + \alpha$ and $\alpha + \beta$ and hence find the value of $(\beta+\gamma)(\gamma + \alpha)(\alpha + \beta)$. 98

12. (a) Solve the equation $x^3 - 9x + 28 = 0$ by using cardon's method. 103

(b) Remove the second term of the equation :

$$x^4 + 8x^3 + x - 5 = 0$$ 102

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Rekha V.V.I. Questions for 2023 Examination

Answer of below mentioned V.V.I. questions are present in your Rekha Examination Guide and Guess Part-I Maths-2

Answer any six questions

1. (a) Find the condition that the line $lx + my + n = 0$ may touch the conic $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. **V. V. I.** 9

   (b) Find the equation of pair of tangents from (x_1, y_1) to the conic : $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c= 0$ **V. V. I.** 10

2. (a) Obtain the polar equation of a conic in the standard form $l/r = 1 + e\cos\theta$. **V. V. I.** .. 22

   (b) Find the equation of the tangent to the conic $1/r = 1 + e \cos\theta$ at the point whose vectorial angle is α. **V. V. I.** 24

   (c) Show that the locus of the pole of a given straight line with respect to a system of confocal conics is a straight line which is normal to that confocal which the straight line touches. 38

3. (a) If the normal at L, one of the extremities of the latus rectum of the conic $l/r = 1 + e\cos\theta$, meet the curve again at P, show that $\frac{\ell}{sp} = \frac{1+e^2-e^4}{1+3e^2-e^4}$. **V. V. I.** ...34

   (b) Obtain equation of normal to conic $\frac{l}{r} = 1 + l\cos\theta$ at the point whose vectorial angle is α. **V. V. I.** 24

4. Show that the equation of the director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$ is $r^2(1-e^2) + 2e\, lr\cos - 2l^2 = 0$ 39

5. (a) Find the condition that the general homogeneous equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ of the second degree in x, y, z should represent two planes and find the angle between them. **V. V. I.** 44

   (b) Prove that $\frac{a}{y-z} + \frac{b}{z-x} + \frac{c}{x-y} = 0$ represents a pair of planes. **VVI** 46

6. (a) Prove that the four planes $my + nz = 0$, $nz + lx = 0$, $lx + my = 0$ and $lx + my + nz = p_3$ form a tetrahedron whose volume is $\frac{p}{lmn}$. 56

   (b) Find the equation of the plane in terms of the intercepts which it makes on the axes. 43

7. (a) Show that the shortest distance between any two opposite edges of the tetrahedron formed by planes $y + z = 0$, $z + x = 0$, $x + y = 0$, $x + y + z = a$ is $\frac{2a}{\sqrt{6}}$. **V. V. I.** 52

MATHS - 2 (Hons.) (2022)

1. (a) Prove that every cartesian equation of second degree represents a conic. 7

(b) What conic section is represented by $2x^2+3y^2-4x-12y+13=0$? 17

2. (a) Prove that two confocal conies intersects each other at right angle. 36

(b) Prove that the locus of the pole of a given straight line with respect to a series of confocal conics is a straight line. 37

3. (a) Define a confocal system of conics. Prove that through every point in the plane of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, two confocal conics can be drawn, one an ellipse and the other a hyperbola. 35

(b) Obtain the equation of the director circle of the conic $\frac{l}{r}=1+e\cos\theta$. 22

4. (a) Find those diameters of the conic $S=x^2+4xy+y^2-2x-2y-6=0$ which touch the parabola $y^2=8x$.

(b) If normal at α, β, γ on parabola $\frac{1}{r}=1+\cos\theta$ meet at point (ρ, ϕ). Prove that $2\phi=\alpha+\beta+\gamma$.

5. (a) Prove that the general equation of first degree equation is x, y, z represents a plane. 42

(b) Find the volume of the tetrahedron whose vertices are (x_r, y_r, z_r), $r = 1, 2, 3, 4$, the axes being rectangular.

6. (a) Find the condition that the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ may lie in the plane $ax+by+cz+d=0$. 46

(b) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. 50

7. (a) Find equation of tangent plane to sphere $x^2+y^2+z^2=a^2$ at any point (x_1, y_1, z_1) on it. 58

(b) Show that the angle between the lines given by $x+y+z=0$ and $ayz+fxz+cxy=0$ is $\frac{\pi}{2}$ if $a+b+c=0$ and $\frac{\pi}{3}$ if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

8. (a) Define a central conicoid. Find the condition that the plane $lx+my+nz=p$ may touch the conicoid $ax^2+by^2+cz^2=1$.

(b) A tangent plane to the ellipsoid $\frac{x^2}{a^z}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the coordinate axes in points A, B and C. Prove that the centroid of the triangle ABC lies on the focus of $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9.$

9. (a) Expand sin θ in ascending powers of θ. 97

(b) Using De Moivre's theorem solve the equation $x^7+x^4+x^3+1=0$

10. (a) Prove that $\log(\alpha+i\beta)=\frac{1}{2}\log(\alpha^2+\beta^2)+i\tan\frac{\beta}{\alpha}$ 100

(b) Express $(\alpha+i\beta)^{x+iy}$ in the form $A+iB$. 101

11. (a) Find the sum of cosines of n angles which are in A.P.

(b) Sum to n terms the series

$$\tan^{-1}\frac{1}{1+1+1^2}+\tan^{-1}\frac{1}{1+2+2^2}+\tan^{-1}\frac{1}{1+3+3^2}+---$$ 113

12. (a) Express sin θ as on infinite product. 124

(b) If $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ then show that $\tan\frac{u}{2}=\tan\frac{\theta}{2}.$

MATHS - 2 (Hons.) (2021)

Answer any six questions.

1. (a) Find the condition that the line lx + my + n = 0 may touch the conic $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$. 9

(b) Prove that the sum of the ordinates of the feet of all the normals drawn from an external point to the parabola is equal to zero.

2. (a) What conic is represented by $2x^2-24xy+16y^2-18x-101y+19=0$? Reduce the equation to standard form and find the latus rectum of the conic.

(b) Prove that the locus of the foot of the perpendicular from the focus of a parabola on the tangent at any point is the tangent at the vertex.

3. (a) Obtain the polar equation of a conic in the standard form $l/r = 1+e\cos\theta$. 22

(b) Find the equation of the tangent to the conic $1/r = 1 + e\cos\theta$ at the point whose vectorial angle is α. 24

4. (a) If the normal at L, one of the extremities of the latus rectum of the conic $l/r = 1 + e\cos\theta$, meet the curve again at P, show that $\frac{\ell}{sp}=\frac{1+e^2-e^4}{1+3e^2-e^4}$. 34

(b) Prove that the locus of the point of contact of the tangents from a given point to a system of confocals is a cubic curve which passes through the given point and through the foci.

5. (a) Find the condition that the general homogeneous equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ of the second degree in x, y, z should represent two planes and find the angle between them. 44

(b) Prove that $\frac{a}{y-z} + \frac{b}{z-x} + \frac{c}{x-y} = 0$ represents a pair of planes. 46

6. (a) Prove that the plane through the point (α, β, γ) and the line $x = py + q = rz + s$ is given by $\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & ry+s \\ 1 & 1 & 1 \end{vmatrix} = 0$

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by planes $y + z = 0$, $z + x = 0$, $x + y = 0$, $x + y + z = a$ is $\frac{2a}{\sqrt{6}}$. 52

7. (a) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ at any point (x_1, y_1, z_1) on it.

(b) Find the locus of points from which three mutually perpendicular lines can be drawn to intersect the conic $ax^2 + by^2 = 1, z = 0$.

8. (a) Find the condition that the plane $lx + my + nz = p$ should be a tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. 75

(b) Prove that six normal can be drawn from an external point to an ellipsoid. 80

9. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$. Prove that $\Sigma \sin 2\alpha = \Sigma \cos 2\alpha = 0$ and $\Sigma \sin^2\alpha = \Sigma \cos^2\alpha = 3/2$.

(b) Expand $\cos\theta$ in ascending powers of θ.

10. (a) If $x = \log\tan\left(\frac{\pi}{4} + \frac{y}{2}\right)$. Prove that $y = -i \log\tan\left(\frac{ix}{2} + \frac{\pi}{4}\right)$. 96

(b) State and prove Gregory's series. 118

11. (a) If $\cos^{-1}(u + iv) = \alpha + i\beta$, where u, v, α and β are all real. Prove that $\cos^2\alpha$ and $\cos^2\beta$ are the roots of the equation $x^2 - (1 + u^2 + v^2)x + u^2 = 0$.

(b) Sum to n terms of the series $\cos\theta + \cos 3\theta + \cos 5\theta + ---$ to n terms, and with help of it prove that $1^2 + 3^2 + 5^2 + ---$ to n terms $= \frac{n(2n-1)(2n+1)}{3}$.

12. (a) Sum the series $1+\frac{\cos 4\theta}{\lfloor 4}+\frac{\cos 8\theta}{\lfloor 4}+---\infty$.

(b) Prove that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+---$ to infinity $=\frac{\pi^2}{6}$.

MATHS - 2 (Hons.) (2020)

Answer any six questions

1. (a) Prove that every Cartesian equation of second degree represents a conic section. 7

(b) Prove that in general a straight line cuts a conic in two points real of imaginary.

2. Trace the conic $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$.

3. (a) Define a confocal system of conics. Prove that through every point in the plane of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ two confocal coincs can be drwon, one an ellipse and the other a hyperbola. 35

(b) Show that the locus of the pole of a given straight line with respect to a system of confocal conics is a straight line which is normal to that confocal which the straight line touches. 38

4. (a) Obtain equation of normal to conic $\frac{l}{r} = 1 + l\cos\theta$ at the point whose vectorial angle is α. 24

(b) If normals at α, β, γ on parabola $\frac{l}{r} = 1 + \cos\theta$ meet at point (ρ,ϕ), prove that $2\phi = \alpha + \beta + \gamma$.

5. (a) Prove that the general equation of first degree equation in x, y, z represents a plane. 42

(b) Prove that the four planes $my + nz = 0$, $nz + lx = 0$, $lx + my = 0$ and $lx + my + nz = p_3$ form a tetrahedron whose volume is $\frac{p}{lmn}$. 56

6. (a) Find the condition that the two lines $\frac{x-\alpha_1}{l_1}=\frac{y-\beta_1}{m_1}=\frac{n-\gamma_1}{n_1}$ and $\frac{x-\alpha_2}{l_2}=\frac{y-\beta_2}{m_2}=\frac{n-\gamma_2}{n_2}$ may be coplanar. 49

(b) Find the length and equations of the line of shortest distance between lines $\frac{x+3}{-4}=\frac{y-6}{3}=\frac{z}{2}$ and $\frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$

7. (a) Find equation of tangent plane to sphere $x^2 + y^2 + z^2 = a^2$ at any point (x_1, y_1, z_1) on it. 58

(b) Show that the angle between the lines given by $x + y + z = 0$ and $ayz + fzx + cxy = 0$ is $\frac{\pi}{2}$ if $a + b + c = 0$ but $\frac{\pi}{3}$ if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

8. (a) Find equation of normal at point (x_1, y_1, z_1) on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(b) If the axes be rectangular, find the locus of equi-conjugate diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 57

9. (a) Expand $\sin\theta$ in ascending powers of θ. 97

(b) Using De Moiver's theorem solve the equation $x^7 + x^4 + x^3 + 1 = 0$.

10. (a) Prove $\log(\alpha + i\beta) = \frac{1}{2}\log(\alpha^2+\beta^2) + i\tan^{-1}\frac{\beta}{\alpha}$. 100

(b) if ${}_{i^i} i^{\text{to}\infty} = \alpha + i\beta$, then prove that $\tan\frac{\pi\alpha}{2} = \frac{\beta}{\alpha}$ and $\alpha^2+\beta^2=e^{-\pi\beta}$. 101

11. (a) Find the sum of cosines of n angles which are in A. P.

(b) Sum to n terms the series $\tan^{-1}\frac{1}{1+1+1^2} + \tan^{-1}\frac{1}{1+2+2^2} + \tan^{-1}\frac{1}{1+3+3^2} + \ldots\ldots$ 113

12. (a) Express $\cos\theta$ as an infinite product. 126

(b) Show $2i\tan^{-1}\left\{i\tan\left(\frac{\pi}{4}-\theta\right)\right\} = \log e \tan\theta$.

MATHS - 2 (Hons.) (2019)

Answer any six questions

1. (a) Find the equation of pair of tangents from (x_1, y_1) to the conic : $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ 10

(b) What conic section is represented by $2x^2 + 3y^2 - 4x - 12y + 13 = 0$? 17

2. (a) Prove that two confocal conics intersects each other at right angle. 36

(b) Prove that one and only one conic of a confocal system will touch a given straight line. 37

3. (a) Find the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point whose vectorial angle is α. 24

(b) If the normal is drawn at one extremity of the latus rectum of the conic $\frac{l}{r} = 1 + e\cos\theta$, prove that the distance from the locus of the other point in which it meets the conic is $\left(\frac{1+3e^2+e^4}{1+e^2+e^4}\right)l$. 34

4. Show that the equation of the director circle of the conic $\frac{l}{r} = 1 + e\cos\theta$ is $r^2(1-e^2) + 2e\,lr\cos - 2l^2 = 0$ 39

5. (a) Find the equation of the plane in terms of the intercepts which it makes on the axes. 43

(b) Prove that the points (0, –1, 0), (2, 1, –1), (1, 1, 1) and (3, 3, 0) are coplanar.

6. (a) Find the shortest distance between the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{n-\gamma}{n} \text{ and } \frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{n-\gamma'}{n'}$$ 51

(b) If the length of two opposite edges of a tetrahedron are a, b; their shortest distance is equal to d and the angle between them is θ, then prove that the volume is $\frac{1}{6}abd\sin\theta$. 57

7. (a) Prove that thespheres : $S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and $S' = x^2 + y^2 + z^2 + 2u'x + 2v'y + 2wz + d' = 0$ will intersect orthogonally, if $2uu' + 2vv' + 2ww' = d + d'$. 68

(b) Two spheres of radii r_1 and r_2 cut each other orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$. 67

8. (a) Find the condition when the plane $lx + my + nz = p$ becomes a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$. 75

(b) Find the condition that a cone may have three mutually perpendicular generators. '. 76

9. (a) State and prove De Moivre's theorem for rational index. 88

(b) If $x_r = \cos\frac{\pi}{2^r} + l\sin\frac{\pi}{2^r}$ prove that $x_1 . x_2 . x_3$ ----- to $\infty = -1$. 91

10. (a) State and prove Gregory's series. 118

(b) If $A + iB = \log(x + iy)$, then prove that $A = \frac{1}{2}\log(x^2 + y^2)$ and $B = \tan^{-1}\frac{y}{x}$ 100

11. (a) Find the sum of sines of n angles which are in A. P. 108

(b) Find the sum of the series : $\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta .. \infty$ 116

12. (a) Resolve sin e as an infinite product 124

(b) If $u = \log\tan\left(\frac{\pi}{4} + \frac{1}{2}\theta\right)$ then show that $\tan\frac{1}{2}u = \tan\frac{1}{2}\theta$.

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